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SURVEILLANCE/POUNCE MODEL

by

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A force of mobile targets is subject to a long period of surveillance, followed by a sudden application of force (the "pounce"). Some targets will escape the pounce because they have not been recently enough localized by the surveillance system. The problems considered are the division of a budget between surveillance and pounce, the allocation of pounce forces to targets, and the description of how the fraction of targets that survives the pounce depends on the budget.

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Abstract

A force of mobile targets is subject to a long period of surveillance, followed by a sudden application of force (the "pounce"). Some targets will escape the pounce because they have not been recently enough localized by the surveillance system. The problems considered are the division of a budget between surveillance and pounce, the allocation of pounce forces to targets, and the description of how the fraction of targets that survives the pounce depends on the budget.

Surveillance/Pounce Model

1. Introduction

It is sometimes imagined that a major war might begin, and having begun might end, with a sudden attack by one side on the strategic nuclear retaliatory forces of the other. There are two properties of the target forces that might enable them to survive such an attack, and therefore hopefully to deter it in the first place: hardness and mobility. Hardness can be overcome by accuracy, whereas mobility cannot. Since accuracy seems to be destined to increase with time, it is natural to study the vulnerabilities of mobile systems. That is the purpose of this report. More specifically, the purpose is to study how the survivability of a mobile force can be expected to depend on the budget devoted to reducing it, and on how that budget ought to be spent, given the surveillance/pounce attack structure that is outlined more precisely below.

The attack cannot begin in complete ignorance of the locations of the mobile targets if it is to have any significant prospect of success. The surface area of the earth is very large in comparison to the lethal area of even a nuclear weapon. This remains true even if a priori considerations reduce the operating area to "only" a desert or an ocean. If the the attack is to be a surprise attack, it seems safe to conclude that a surveillance system of some sort must operate during the period preceding the surprise, having the function of continuously providing localization information that will be utilized by the pounce system when the attack is made. Since both systems contribute to the same final goal, the attacker must face the budgeting problem mentioned above.

Surveillance systems can be categorized in various ways, amongst which are saturability (the number of targets that can be handled simultaneously), accuracy, and timeliness. The system to be studied here is insaturable and perfectly accurate, being imperfect only in the respect that position reports on targets occur occasionally, rather than continuously. Spending money on the system simply increases the frequency of contacts. One might imagine launching more satellites, hiring more agents, or implanting more sensors in the operating area.

Let T_i be the age of the most recent contact on the $i\underline{^{th}}$ target at the time of the attack. If the time late T_i is large, then a large amount of of pounce force will be required to make up for the large associated uncertainty area, or alternatively the target can be allowed to escape and the pounce forces concentrated on easier targets. In other words, there is a programming problem involved: given the times late for all the targets, how should the pounce force be distributed amongst the targets in order to minimize the average number of targets that survive the pounce? It is only after this programming problem has been solved that the tradeoff between surveillance and pounce forces can be considered, and finally the relationship between survival probability and total budget can be determined. Most of this essentially mathematical activity is carried out in Section 4. Section 3 summarizes the major points and illustrates the relationship between survival probability and total budget with an example.

2. Assumptions, definitions, and notation

Contacts on each target are assumed to be independent Poisson processes, with the contact rate λ being a linear function of the amount of money spent on the surveillance system. The mean time between contacts on each target is therefore $1/\lambda$. The results obtained below would probably not be qualitatively different if detections occurred regularly on each target every $1/\lambda$, provided the phases of the contact streams were all independent. The Poisson assumption is natural if contacts are due to irregular, localized phenomena that make the targets occasionally visible to the surveillance system. The time of the attack is assumed to be independent of the surveillance process; i.e., the attacker does not have the option of waiting until all of the times T_i happen to be small (as will happen eventually) before attacking. The late times T_i are therefore all independent and exponentially distributed with mean $1/\lambda$.

If A_i pounce units are allocated to the i^{th} target, the probability that target i survives is assumed to be $\exp(-A_i/kT_i)$, where k is some constant. There are at least two sets of assumptions where this formula is appropriate:

a) If the target's motion is diffusion in two dimensions [1], then the expression is correct if A_i represents lethal area (in the shape of a circle with radius $\sqrt{A_i/\pi}$) and k is 2π times the diffusion constant ($kt/2\pi$ is the variance of target position relative to the last point of contact in each dimension). In this case, the pounce forces consist of a certain number of weapons, with the translation from weapons to lethal area depending on the hardness of the targets.

b) If each target has a top speed v, and if each target is aware of any contacts made on it, and if the pounce force essentially has the job of re-establishing contact by area search of an expanding circle [2], then the formula applies with $k = \pi v^2$ and A_i being the search rate (area per unit time) of all assigned pounce forces, assuming that the amount of time available for area search is large compared to $1/\lambda$.

In addition to being applicable in two highly diverse circumstances, the formula $\exp(-A_i/kT_i)$ also has simplifying analytic properties, so it will be employed exclusively in what follows. The sum $A \equiv A_1 + \ldots + A_N$ is fixed and will be called the "total pounce". The cost of A will be assumed linear in A .

The following notation will be employed:

λ = contact rate on each target

c_s = cost of the surveillance system per unit of λ

T_i = time late for target i

A_i = pounce assigned to target i

A = total pounce available

c_p = cost of pounce system per unit pounce

n = number of targets

q = expected fraction of targets surviving

k = constant in the survivability expression $\exp(-A_i/kT_i)$

B = total budget

3. Summary and example

Given n, c_s, c_p, and k, the budget B required to eliminate all but q of the targets can be obtained as follows:

1) Look up the β-value corresponding to q in Figure 1. If $q < .1$, use the formula $\beta = \sqrt{\ln(.655/q)}$ (derived in the appendix).

2) The total budget required is $B = 2\beta\sqrt{n\, c_s\, c_p\, k}$, which should be divided equally between surveillance and pounce forces.

For example, suppose that there are 100 targets that habitually move randomly in such a manner that their motion can be characterized as diffusion, and that k (according to the discussion in Section 1) is 100 mi^2/hr. Suppose further that a surveillance system capable of providing a fix every 10 hours (λ = .1/hr.) on each target would cost $\$10^{11}$, with the cost of better or worse systems being proportional to detection rate;

ie., $c_s = (\$10^{11})/(.1/\text{hr.}) = 10^{12}\$\text{hr}$. Assume that the pounce force is some type of remotely applied weapon for which $c_p = \$10^6/\text{mi}^2$. Then $\sqrt{n\, c_s\, c_p\, k} = \$10^{11}$. If $q = .240$, Figure 1 shows $\beta = 1$, and therefore $\$10^{11}$ should be spent on each system. The result is a surveillance system with λ = .1/hr. and a pounce system with 10^5 mi^2 of lethal area. Doubling the budget would double β, with the consequence that q would be only about .01. Halving the budget would make $q = .56$, etc.

It is remarkable that Figure 1 does <u>not</u> exhibit the exponentially decreasing returns phenomenon that is so common in models of this sort. While it is true that actually wiping out all the targets with certainty is not possible with any finite budget, it is nonetheless possible to obtain remarkably small values for q, as the above example illustrates. Roughly speaking, a surveillance/pounce system either works or it doesn't. If it works at all, then it is likely to be decisive.

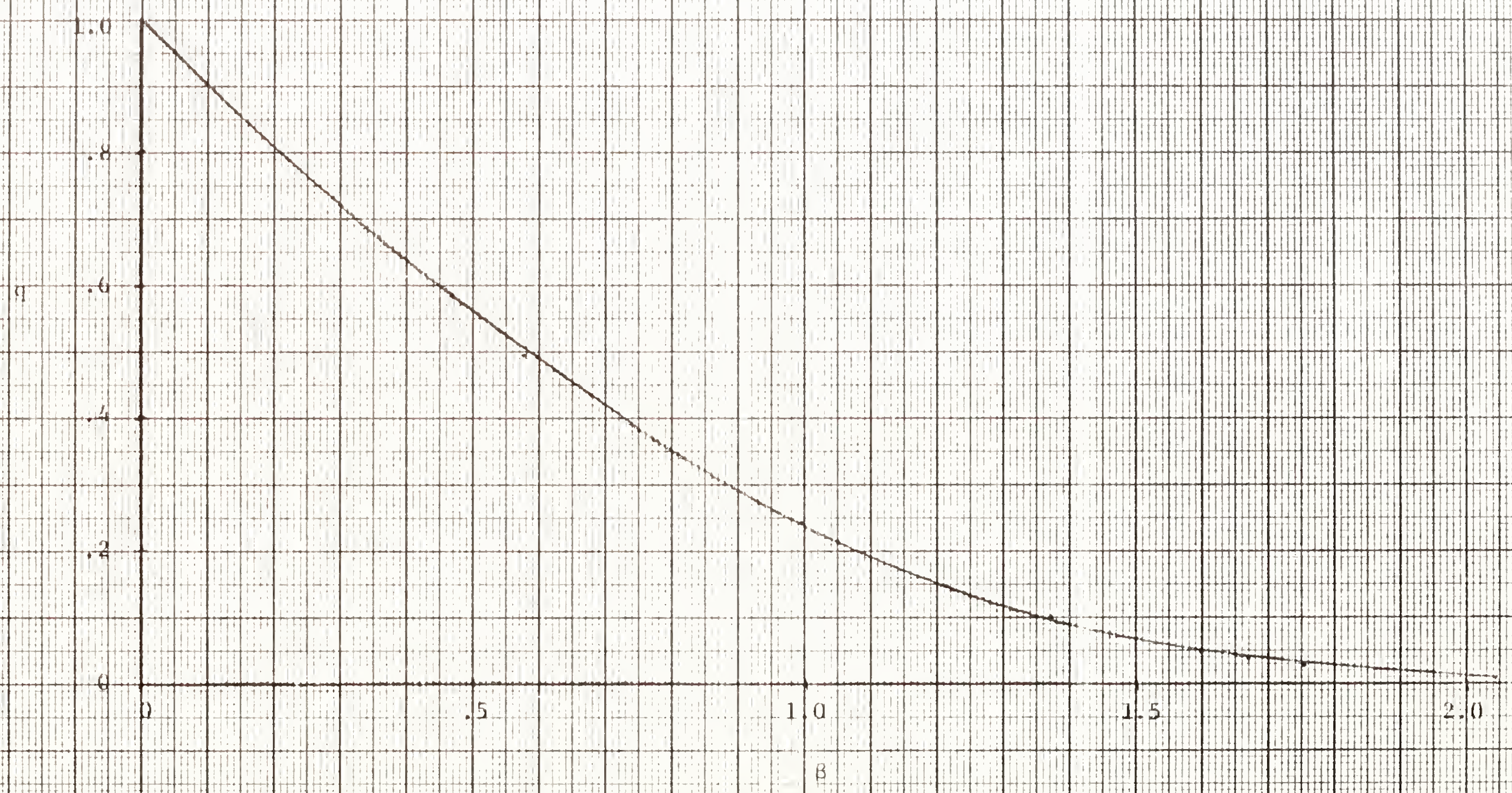

Figure 1

Survival probability q vs. dimensionless budget β

The calculations above were based on the assumption that n is "large". To test the validity of this assumption, a simulation was written wherein $A = 10^5$ units of pounce were optimally allocated to $n = 100$ targets with $\lambda = .1$ and $k = 100$. The optimal allocation program is easy to write for a given A and λ because it is equivalent to an elementary distribution of effort problem in the theory of optimal search [3]. In theory, the average number of surviving targets should be 24.0. After 10,000 replications of the simulation, the estimated mean and standard deviation of the number of survivors were 24.2 and 6, respectively. The difference between 24.0 and 24.2 is small enough to ignore for most purposes, so 100 can be considered "large". Further experimentation revealed that the theoretical model is usually adequate even when n is as small as 10.

4. Analysis for large n

Let $a(u)$ be the amount of pounce allocated to any target for which the time late is u. The probability that a target survives is (averaging over u)

$$(1) \qquad q = \int_0^\infty \lambda \exp(-\lambda u)\exp(-a(u)/ku)du ,$$

and the average amount of pounce per target is

$$(2) \qquad \bar{a} = \int_0^\infty \lambda \exp(-\lambda u)a(u)du$$

The average total pounce required is $n\bar{a}$, and the average cost of the pounce forces is therefore $c_p n \bar{a}$. Since n is assumed large, we will not distinguish between cost and average cost, so the problem of an attacker with a total budget B is to minimize q (as given by (1)) subject to the total cost constraint

$$(3) \qquad c_p n\bar{a} + c_s \lambda \leq B$$

The minimization problem will be solved by introducing a La Grange multiplier $n c_t$ on the survival probability q. The resulting La Grangian expression to be minimized is $L \equiv n c_t q + c_p n \bar{a} + c_s \lambda$. Note that c_t can be interpreted as the damage caused to the attacker by a surviving target, in which case L i "net total loss". In terms of the decision variables $a(\cdot)$ and λ, L is given

$$(4) \qquad L = n \int_0^\infty \lambda \exp(-\lambda u) \{c_t \exp(-a(u)/ku) + c_p a(u)\} du + c_s \lambda$$

Minimization with respect to a(u) can be carried out by equating the derivative of the integrand to 0 and solving for a(u), except that a(u) cannot be negative

The result is

(5) $$a(u) = \begin{cases} ku\,\ell n(u^*/u) & \text{for} \quad u \leq u^* \\ 0 & \text{for} \quad u \geq u^* \end{cases},$$

where $u^* = c_t/(c_p\,k)$. Note that $a(0) = 0$ and $a(u^*) = 0$. The pounce forces are mainly allocated to targets that are neither too easy (in which case a small amount of pounce suffices) nor too hard (in which case no amount will do much good). Substituting (5) into (4), the result is

(6) $$L = n\int_0^{u^*} \lambda\,\exp(-\lambda u)\{c_p\,ku + c_p\,ku\,\ell n(u^*/\mu)\}\,du$$
$$+ n\int_{u^*}^{\infty} \lambda\,\exp(-\lambda u)\{c_t + 0\}\,du + c_s\,\lambda\,.$$

Noting that $n\,c_p ku^* = n\,c_t$, Equation (6) can be rearranged to give

(7) $$L = n\,c_p\,k[\int_0^{u^*} \lambda u\,\exp(-\lambda u)\{1 + \ell n(u^*/u)\}\,du + u^*\,\exp(-\lambda u^*)] + c_s\,\lambda\,.$$

Letting $x = \lambda u^*$ and substituting $v = \lambda u$ in the integrand, (7) is the same as

(8) $$L = n\,c_t(q(x) + b(x)) + x\,c_s/u^*, \text{ where}$$

(9) $$q(x) \equiv (1/x)\int_0^x v\,\exp(-v)\,dv + \exp(-x) = (1 - \exp(-x))/x\,, \text{ and}$$

(10) $$b(x) \equiv (1/x)\int_0^x v\,\exp(-v)\,\ell n(x/v)\,dv\,.$$

The three terms in (8) are the costs of surviving targets, pounce forces, and surveillance forces, respectively.

The object is now to minimize the sum of the three costs (L) with respect to x, since x is simply a dimensionless version of λ. The minimizing x is such that $(d/dx)\,L = 0$, unless c_t is so small that $x = 0$ is optimal. Let $f(x) \equiv -(d/dx)(q(x) + b(x))$. Then $(d/dx)L = 0$ if and only if $f(x) = \alpha$, where $\alpha = c_s/(n\, c_t\, u^*) = (c_s\, c_p\, k)/(n\, c_t^2)$. The function $f(x)$ is discussed further in the appendix. One fact that will prove useful below is that $b(x) = xf(x)$.

The simplest way to investigate the tradeoff between survival probability q and budget B is to use x (the dimensionless surveillance rate) as a parameter. Given x, the survival probability is immediate from the formula for $q(x)$. The cost of surveillance is $x\, c_s/u^*$, the cost of pounce is $n\, c_t\, b(x)$, and B is simply the sum of the two. To determine u^*, one must first obtain c_t from the equation $f(x) = \alpha$; ie.,

$$c_t = [c_s\, c_p\, k/n\, f(x)]^{1/2} \tag{11}$$

It is a remarkable fact that the two costs are always equal, with the common value being $(n\, c_s\, c_p\, k\, x^2\, f(x))^{1/2}$. This can be shown by simply substituting into the formulas and using the fact that $b(x) = x\, f(x)$. Therefore $B = 2\sqrt{n\, c_s\, c_p\, k}\,(x\sqrt{f(x)})$. Figure 1 was constructed parametrically by varying x; it shows the tradeoff between survival probability $q(x)$, and the dimensionless budget $x\sqrt{f(x)}$.

APPENDIX

From Section (3), we have

(A1) $$q(x) \equiv (1 - \exp(-x))/x$$

(A2) $$b(x) \equiv (1/x) \int_0^x v \exp(-v)\, \ell n(x/v)\, dv$$

(A3) $$f(x) \equiv (d/dx)(q(x) + b(x))$$

We will first show that $b(x) = xf(x)$. The easiest way to do this is through power series expansion. The power series for $\exp(-x)$ is $\sum_{m=0}^{\infty} (-x)^n/n\,!$ Using this, it is easy to obtain

(A4) $$q(x) = 1 + \sum_{n=1}^{\infty} (-x)^n/(n+1)!$$

The fact that $\int_0^x v^{n+1}\, \ell n(x/v)\, dv = x^{n+2}/(n+2)^2$ can be established by substituting $u = v/x$ and then using the fact that $\int_0^1 u^{n+1}\, \ell n(1/u) = 1/(n+2)^2$. It follows that

(A5) $$b(x) = (1/x) \sum_{n=0}^{\infty} (-x)^{n+2}/n!(n+2)^2 = \sum_{n=1}^{\infty} (-x)^n/(n-1)!(n+1)^2 .$$

Therefore, combining like terms in (A4) and (A5) and using the fact that $1/n - 1/(n+1) = 1/n(n+1)$,

(A6) $$q(x) + b(x) = 1 + \sum_{n=1}^{\infty} (-x)^n/n!(n+1)^2 .$$

Upon taking $-(d/dx)(q(x)+b(x))$ term by term, we obtain

(A7) $$f(x) = \sum_{n=1}^{\infty} (-x)^{n-1}/(n-1)!(n+1)^2 .$$

The fact that $b(x) = xf(x)$ follows upon comparing terms in (A5) and (A7).

The power series (A7) can conveniently be used to compute $f(x)$ when x is small. For large x, however, an approximation can be based on the fact that

(A8) $$f(x) = (1/x^2)\left\{\int_0^x q(t)dt + \exp(-x) - 1\right\} ,$$

which is most easily demonstrated by showing that the power series of the right hand side of (A8) is the same as (A7). Since

(A9) $$\lim_{x\to\infty} \int_0^x q(t)dt - \ln x = \gamma , \quad \text{where}$$

$\gamma = .577 \ldots$ is Euler's constant [3], this leads to the approximation

(A10) $$\hat{f}(x) = (1/x^2)(\gamma + \ln x - 1)$$

Since $\gamma - 1 = \ln(.655)$, (A10) can also be expressed as

(A11) $$x^2\hat{f}(x) = \ln(.655x)$$

Finally, since $q(x) \simeq 1/x$ for large x, the expression given in Section 3 is simply $x\sqrt{\hat{f}(x)}$ with $1/q$ substituted for x.

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2] Washburn, A. R. (1981), Search and Detection, Operations Research Society of America, pp. 2-8 to 2-11.

3] Washburn, A. R. (1981), Search and Detection, Operations Research Society of America, pp. 5-3 to 5-6.

4] Standard Mathematical Tables (12th ed.), definite integral No. 458, Chemical Rubber Co., Cleveland.

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